MATHEMATICS 3C/3D

Section One: Calculator-free

Please place your student identification label in this box

Student Number: In figures

In words _____

_____

Number of additional
answer booklets used
(if applicable): _____

Time allowed for this section
Reading time before commencing work: five minutes
Working time for section: fifty minutes

Materials required/recommended for this section
To be provided by the supervisor
This Question/Answer Booklet
Formula Sheet

To be provided by the candidate
Standard items: pens (blue/black preferred), pencils (including coloured), sharpener,
correction tape/fluid, eraser, ruler, highlighters

Special items: nil

Important note to candidates
No other items may be taken into the examination room. It is **your** responsibility to ensure that
you do not have any unauthorised notes or other items of a non-personal nature in the
examination room. If you have any unauthorised material with you, hand it to the supervisor
before reading any further.

Structure of this paper

Section	Number of questions available	Number of questions to be answered	Working time (minutes)	Marks available	Percentage of total exam
Section One: Calculator-free	8	8	50	50	$33\frac{1}{3}$
Section Two: Calculator-assumed	13	13	100	100	$66\frac{2}{3}$
			Total	150	100

Instructions to candidates

1.   The rules for the conduct of Western Australian external examinations are detailed in the *Year 12 Information Handbook 2012*. Sitting this examination implies that you agree to abide by these rules.

2.   Write your answers in the spaces provided in this Question/Answer Booklet. Spare pages are included at the end of this booklet. They can be used for planning your responses and/or as additional space if required to continue an answer.
   • Planning: If you use the spare pages for planning, indicate this clearly at the top of the page.
   • Continuing an answer: If you need to use the space to continue an answer, indicate in the original answer space where the answer is continued, i.e. give the page number. Fill in the number of the question(s) that you are continuing to answer at the top of the page.

3.   **Show all your working clearly.** Your working should be in sufficient detail to allow your answers to be checked readily and for marks to be awarded for reasoning. Incorrect answers given without supporting reasoning cannot be allocated any marks. For any question or part question worth more than two marks, valid working or justification is required to receive full marks. If you repeat an answer to any question, ensure that you cancel the answer you do not wish to have marked.

4.   It is recommended that you **do not use pencil**, except in diagrams.

5.   The Formula Sheet is **not** handed in with your Question/Answer Booklet.

Section One: Calculator-free (50 Marks)

This section has **eight (8)** questions. Answer **all** questions. Write your answers in the spaces provided.

Spare pages are included at the end of this booklet. They can be used for planning your responses and/or as additional space if required to continue an answer.
- Planning: If you use the spare pages for planning, indicate this clearly at the top of the page.
- Continuing an answer: If you need to use the space to continue an answer, indicate in the original answer space where the answer is continued, i.e. give the page number. Fill in the number of the question(s) that you are continuing to answer at the top of the page.

Suggested working time: 50 minutes.

Question 1 (4 marks)

Let $f(x) = (x+3)(1-x^2)^5$.

The derivative of $f(x)$ can be written in the form $f'(x) = (1-x^2)^4(ax^2+bx+c)$.

Determine the values of a, b and c.

Question 2 **(5 marks)**

A company made 16 motorbikes of three different types.

Each type A motorbike cost $5000 to make, while each type B motorbike cost $2000 and each type C cost $1000. The company spent $65 000 making the 16 motorbikes.

The number of type A motorbikes made was three times the total number of type B and C motorbikes.

Let a = number of type A motorbikes,
b = number of type B motorbikes, and
c = number of type C motorbikes.

Some of the information above is represented by the two equations:

$$a + b + c = 16$$
$$5a + 2b + c = 65$$

(a) Write down a third equation which, together with the equations above, is sufficient to determine the values of a, b and c. (1 mark)

(b) How many of each type of motorbike were made? (4 marks)

Question 3   **(7 marks)**

Let A, B, C, D, E, F and G be points on the graph of a continuous function $f(x)$.
The table below contains information about the sign of $f(x)$, $f'(x)$ and $f''(x)$ at these points.
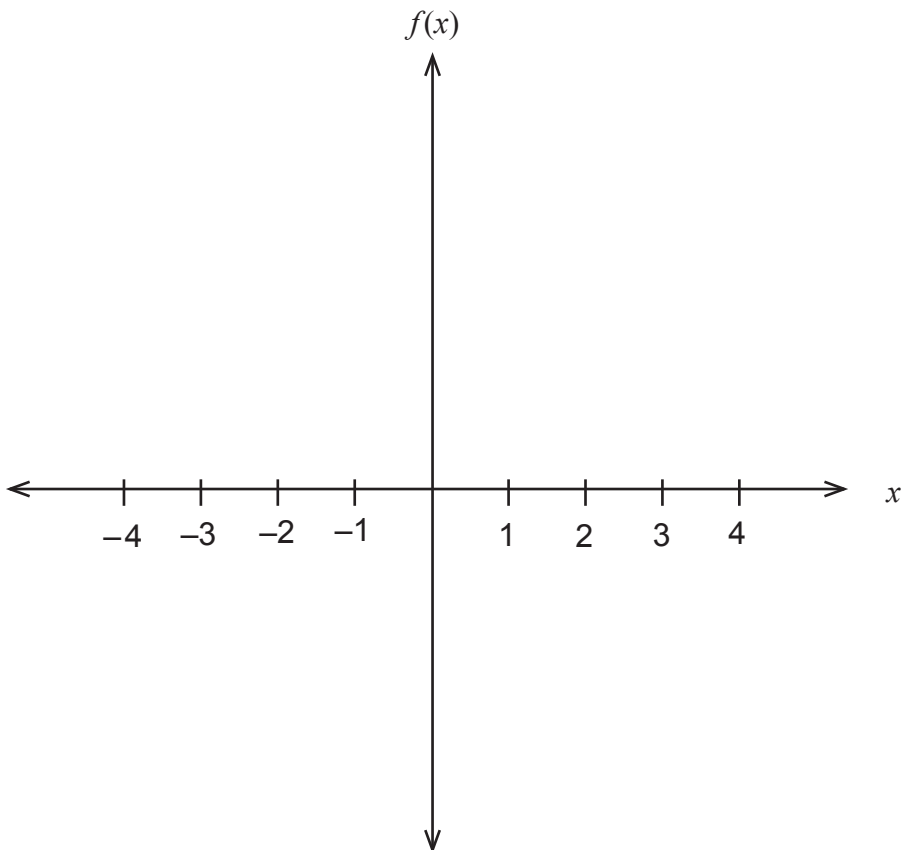
Point	A	B	C	D	E	F	G
x	−4	−3	−1	0	1	2	4
$f(x)$	+	0	−	0	+	+	+
$f'(x)$	−	−	0	+	+	0	+
$f''(x)$	+	+	+	0	−	0	+

There are no other points at which $f(x)$, $f'(x)$ or $f''(x)$ are equal to zero.

(a)   Which point is a local minimum?   (1 mark)

(b)   Describe the nature of the graph at point F.   (2 marks)

(c)   Sketch the function on the axes below.   (4 marks)

$f(x)$

x

−4   −3   −2   −1   1   2   3   4

See next page

Question 4 **(7 marks)**

Two events A and B have the following properties.

$P(A \cup B) = 0.8$
$P(A \cap B) = 0.3$
$P(A) = 0.6$

(a) Calculate:

(i) P(B). (1 mark)

(ii) $P(\overline{A} \cap B)$. (2 marks)

(b) For a third event C, $P(C \mid B) = 0.4$.

(i) Calculate $P(B \cap C)$. (1 mark)

(ii) If events B and C above are independent, and events A and C are mutually exclusive, determine the value of $P(A \cup C)$. (3 marks)

See next page

Question 5 **(9 marks)**

(a) Evaluate $\int_0^1 8x\,(2x^2-1)^7 dx$. (3 marks)

(b) If $\dfrac{dy}{dx} = \dfrac{2}{x^2} + 4x$, and $y = 3$ when $x = 2$, determine the value of y when $x = 5$. (3 marks)

(c) Evaluate $\int_1^2 \dfrac{d}{dx}\left(\dfrac{x^3}{x^2+1}\right) dx$. (3 marks)

Question 6   **(6 marks)**

(a)   Express $\dfrac{5}{x+5} - \dfrac{2}{x+2}$ in the form $\dfrac{ax+b}{(x+5)(x+2)}$, where a and b are constants.
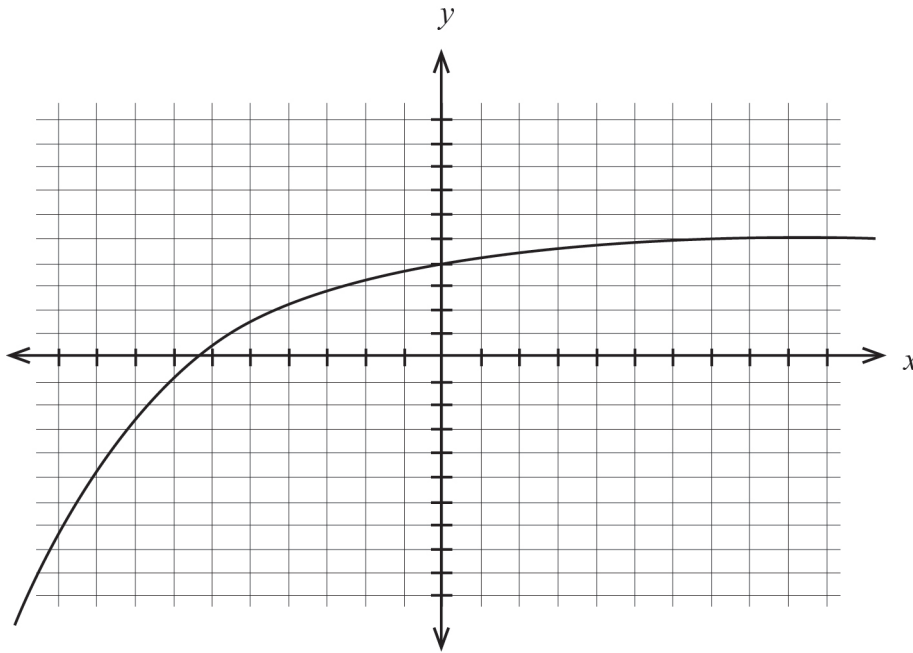
                                                                                            (2 marks)

(b)   Using your answer to Part (a) or otherwise, solve the inequality $\dfrac{5}{x+5} > \dfrac{2}{x+2}$. (4 marks)

See next page

Question 7 **(6 marks)**

Part of the graph of $y = a + be^{cx}$, where a, b and c are constants, is shown below.

(a) Which of the constants a, b and c are positive, and which are negative? Justify your answers. (3 marks)

(b) Sketch on the same axes the graph of $y = -a - be^{-cx}$. (3 marks)

See next page

Question 8 **(6 marks)**

A continuous function $f(x)$ is increasing on the interval $0 < x < 3$ and decreasing on the interval $3 < x < 6$. Some of its values are given in the table below.

x	0	1	2	3	4	5	6
$f(x)$	5	16	27	32	25	0	-49

The function $F(x)$ is defined, for $0 \leq x \leq 6$, by $F(x) = \int_0^x f(t)dt$.

(a)   At which value of x in the interval $0 \leq x \leq 6$ is $F(x)$ greatest? Justify your answer.

(2 marks)

(b)   At which value of x in the interval $0 \leq x \leq 6$ is $F'(x)$ greatest? Justify your answer.

(2 marks)

See next page

(c)   Use the values of $f(x)$ in the table to show that $48 \leq F(3) \leq 75$.   (2 marks)

End of questions

Additional working space

Question number: _____

Additional working space

Question number: _____

Additional working space

Question number: _____

Additional working space

Question number: _____